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Abstract

Empirical evidence on developing countries shows on the one hand that rich farm-households are more keen to adopt new technologies and are higher risk takers than poor households. On the other hand, however, they are shown to be less vulnerable to income shocks than poor farmers. This paper provides a rationale for these observations. Risk averse agents, heterogeneously endowed with wealth, non-cooperatively decide on their level of subscription to risk-sharing and on the degree of individual production risk they take. Rich households take more risks and subscribe more to risk-sharing. Although risk-sharing allows all households to cope with idiosyncratic shocks, the risk-taking behavior of rich households increases the covariate component of poor households' income variance through risk-sharing, deterring the participation of the poor. These poor households in turn opt for safer but less productive production plans.

JEL Codes: O12, O13, O17, O33

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1 Introduction

In developing countries, the ability of farm households to deal with risk is a key determinant of their daily livelihood. Poverty, food security and insurance issues are intrinsically linked to one another. Due to asymmetric information, moral hazard and lack of infrastructure, the market fails to provide efficient insurance and credit services. Depending on their nature, households try to mitigate the effects of uncertainty by making use of various mechanisms. On the one hand, the magnitude of income shocks depends on the level of risk-taking resulting from the household's production decisions. On the other hand, informal insurance transfers between households allow to share and absorb risk ex post. As intuition suggests, ex ante risk-management and ex post risk-coping decisions should be interrelated. For instance, by a classical moral hazard argument, the level of risk-sharing achieved in informal insurance networks should have an impact on the households' risk-taking behavior. However, to the best of our knowledge, risk-taking and risk-sharing tend to be analyzed as distinct topics[1]. This paper aims at exploring the interaction between risk-taking decisions and subscription to informal risk-sharing under wealth heterogeneity. The analysis of this relationship provides a rationale for two important stylized facts. First, household wealth is an important determinant of risk-taking in general (Dercon (1996), Dercon (1998)) and technology adoption (Dercon & Christiaensen (2011)), in particular, which tend to be low among the poorest households. Second, empirical evidence suggests that poor households' consumption path is more affected by idiosyncratic shocks (Jalan & Ravallion (1999); Morduch (1995)). Our model predicts a risk-taking profile that is consistent with the former observation, while the latter can be credibly explained by our second important theoretical prediction, according to which subscription to risk-sharing is positively related to household wealth. We highlight that this pattern of involvement in risk-sharing within informal insurance groups is the outcome of an externality in terms of risk: we show that subscription to risk-sharing entails an exposure to the other participants' risk-taking behavior.

The strategies that a farm household can adopt to deal with income risk can be divided into two categories, ex ante and ex post, depending on whether they are implemented before or after the income draw.

As far as ex ante strategies are concerned, household can affect the distribution and magnitude of shocks by adapting their production plans and risk taking strategy. This type of behavior is known as risk management (Dercon (2002)). The diversification of income sources is a first solution, both in terms of economic activities (agricultural and non-agricultural sectors) and geographical locations (urban and rural environments) (Morduch (1995), Sarpong & Asuming-Brempong (2004)). A second mechanism pertains to technology adoption and production choices, which can entail lower risks at the expense of lower expected returns. Kurosaki & Fafchamps (2002) provide empirical evidence that production choices are strongly affected by risk-management considerations in contexts of market imperfections.

Ex post mechanisms are aimed at absorbing a given income shock, that is reducing its impact on consumption.

At the household level, three important examples can be mentioned. First, households facing shocks can sell assets used as buffer stocks such as cattle (McPeak (2004), Kazianga & Udry (2006) and Verpoorten (2009)). Second, household adjustments in terms of composition and activities can be made, such as the

[1] A potential relationship between risk-sharing and technology adoption is however mentioned by Bandiera & Rasul (2006). They highlight an inverted-U shaped relationship between an individual farmer's propensity to adopt a new crop and the number of adopters in her social network. Among the potential explanations for the existence of a negative partial effect of the rate of adoption in the network on individual propensity to adopt, the authors mention the fact that a high rate of adoption reduces the scope for risk-sharing. However, this argument relies more on income correlations than on risk-taking.

use of child fostering (Akresh (2009)) and child labor. Björkman (2006) finds that income shocks have large negative and highly significant effects on female enrollment in primary schools and the effect grows stronger for older girls in Uganda. Using panel data from Tanzania, Beegle et al. (2008) find that households respond to transitory income shocks by increasing child labor, but that the extent to which child labor is used as a buffer is lower when households have access to credit. Using panel data from Madagascar, Gubert & Robilliard (2008) find that transitory income shocks have a significant impact on the probability of leaving school. Jacoby & Skoufias (1997) study responses to aggregate and idiosyncratic, as well as to anticipated and unanticipated, income shocks in India. They find that seasonal fluctuations in school attendance are a form of self-insurance but one which does not result in a substantial loss of human capital on average. Third, households can modify the structure of their consumption patterns, reducing for instance non-food expenditures.

At the community level, households can engage in risk-sharing mechanisms. Subscription to risk-sharing, while decided before, is an important ex post risk-coping tool since income transfers allow to mitigate the transmission of a shock to consumption. From a theoretical point of view, limited commitment has been highlighted as a major constraint to risk-sharing (Coate & Ravallion (1993); Kocherlakota (1996); Ligon et al. (2002); Murgai et al. (2002)). Empirically, the hypothesis of complete risk-pooling is also often rejected (Townsend (1994); Jalan & Ravallion (1999); Hoogeveen (2002); Murgai et al. (2002); Morduch (1995)), even within nuclear households (Duflo & Udry (2003)). By distinguishing between food and non food outcomes, De Weerdt & Dercon (2006) reach, however, a more subtle conclusion since they do not reject full insurance at the community level when the focus is on food consumption.

In this paper, it is shown that wealth heterogeneity can also produce a pattern of incomplete risk-pooling, even if perfect commitment is assumed.

Our contribution consists of an attempt to integrate the most important informal insurance mechanisms in a unified framework. In the following sections, we develop a model in which members of an insurance network non-cooperatively decide on the extent of their risk-taking and on their subscription to risk-sharing. In Section 2, the risk-taking decision is analyzed in autarky, that is in the absence of risk-sharing. This allows to highlight the basic tradeoff between risk and return. As expected, wealthier agents, whose ability to absorb shocks ex post is higher, are shown take more risk if some mild condition is satisfied. In Section 3, a distinction is drawn between idiosyncratic and covariate shocks. We then introduce the risk-sharing scheme, which proves to be a source of mutual externalities in terms of covariate risk. In Section 4, a benchmark case is presented in which the members of the insurance group are homogeneous in every respect. Risk externalities are shown to generate the classical moral hazard result, in the sense of excessive risk-taking in the non-cooperative equilibrium as compared to the first best. We then turn in Section 5 to the more general case in which the insurance group that we consider is characterized by some degree of wealth heterogeneity. On the one hand, the positive relationship between household wealth and risk-taking appears to be reinforced by the risk-sharing mechanism. On the other hand, the equilibrium profile of participation to risk-sharing is affected by wealth: we show that subscription to risk-sharing is an increasing function of risk-taking and wealth.

2 Risk-taking in autarky

The aim of our modelling strategy is to represent a rural community in which households are faced with different kinds of income shocks. In order to deal with uncertainty, households can make use of three managing and coping strategies that we introduce gradually in the model, namely the extent of their risk-taking in production, their level of subscription to community risk-sharing and their use of buffer stocks. The economy is composed of a continuum of measure H of farm households, or agents, indexed by $h \in [0, H]$. Each agent is endowed with a wealth level w_h.

Let us start by focusing on risk-taking by representing the households' objective in "autarky". Risk-sharing will be introduced in the following section. Incomes are risky, and farm households can affect the shape of this risk by choosing their production plans / risk-taking strategy that we denote by $\sigma \geq 0$. This variable captures the range of production choices that a farm household can take and highlights the following basic tradeoff: more risk-taking raises the expected income, but also increases its variance. Indeed, for a given activity, the farmer can adopt technologies with different implications in terms of risk and return, choosing between traditional and improved seeds, for instance. Besides, the same relationship between risk and return holds in the composition of the household's portfolio of activities: indeed, the household may tend either to specialize, thereby benefiting from economies of scale and learning, or to engage in a wider range of activities in order to diversify its income sources. Formally, agent h's income writes

$$Y_h = \mu(\sigma_h) + S_h, \tag{1}$$

where μ is expected income and has the following properties: (1) $\mu' > 0$, risk-taking increases the expected income, (2) $\mu'' < 0$, with decreasing marginal returns, (3) $\mu(0) \geq 0$, since income cannot be negative. Notice that the latter assumption allows the existence of a risk-free activity.[2]

S_h is a random variable capturing an income shock whose distribution depends on risk taking. It has a conditional mean equal to zero and a conditional variance equal to σ_h^2:

$$\begin{aligned} E(S_h; \sigma_h) &= 0, \\ Var(S_h; \sigma_h) &= \sigma_h^2. \end{aligned}$$

The expected utility of household h writes $Eu(c_h)$, where consumption c_h is based on initial wealth w_h and income Y_h:

$$c_h = w_h + Y_h.$$

Given the distributional assumptions made above, the agent's consumption mean and variance are simply

$$\begin{aligned} E(c_h; \sigma_h) &= w_h + \mu(\sigma_h) \\ VAR(c_h; \sigma_h) &= \sigma_h^2. \end{aligned}$$

Agents are risk averse, so that $u(c)$ is concave and has decreasing absolute risk aversion (DARA). Let $A(c) = -\frac{u''(c)}{u'(c)}$ define the coefficient of absolute risk aversion, with $A' \leq 0$ and $A'' \geq 0$ as is the case in a

[2] It can be easily seen graphically that, taken together, those properties of $\mu(\sigma)$ imply that the average return to risk taking is always larger than the marginal return:

$$\frac{\mu}{\sigma} \geq \mu' \iff \epsilon_{\mu,\sigma} \leq 1,$$

where $\epsilon_{\mu,\sigma} = \mu' \frac{\sigma}{\mu}$ is the elasticity of μ with respect to σ. The latter term will be used later on.

wide range of utility functions, such as the hyperbolic absolute risk aversion (HARA), which encompasses a large class of functions such as CARA, CRRA and quadratic utility functions. The expected utility derived from consumption can be rewritten in the following way:

$$Eu\left(c_h\right) = u\left(C_h\right),$$

where C is the certainty equivalent of the lottery c:

$$C_h = E\left(c_h\right) - \Pi_h,$$

where Π is the risk premium[3]:

$$\Pi_h \approx A\left(E\left(c_h\right)\right)\frac{Var\left(c_h\right)}{2} = A\left(w_h + \mu\left(\sigma_h\right)\right)\frac{\sigma_h^2}{2}.$$

Having introduced the concepts necessary for the analysis of autarky, we can now solve the household's problem. The timing is the following: the household chooses its production plan / risk-taking strategy σ, then shocks and income levels are realized and consumption takes place.

In autarky, the decisions on risk-taking are independent between agents, therefore, for the ease of exposition, we neglect household subscripts for the resolution of this section. Maximizing $Eu(c)$ boils down to maximizing the certainty equivalent C with respect to σ.

$$\max_{\sigma} C \approx w + \mu\left(\sigma\right) - A\left(w + \mu\left(\sigma\right)\right)\frac{\sigma^2}{2}.$$

The optimal level of risk-taking[4] σ^A is obtained by solving the tradeoff between risk and return, that is by equalizing the marginal impact of risk-taking on expected consumption μ' to its marginal effect on the risk premium. This second effect, $\frac{\partial\Pi}{\partial\sigma} = A\sigma + A'\mu'\frac{\sigma^2}{2}$, is composed of two terms of opposite signs. The first, obvious effect of risk-taking on the risk premium is positive and pertains to the increase in consumption variance. The second, negative effect stems from the decrease in absolute risk aversion, A, consecutive to the increase in expected consumption implied by risk-taking. This second term adds up to the first direct effect of increased expected consumption, so that the autarkic level of risk-taking σ^A is implicitly defined by:

$$\mu'\left(\sigma^A\right)\left(1 + |A'|\frac{\left(\sigma^A\right)^2}{2}\right) - A\sigma^A = 0. \tag{2}$$

Let us now analyze the comparative statics of σ^A.

Proposition 1 *In autarky, risk-taking σ^A is increasing in household wealth w if and only if*

$$\frac{\partial^2\Pi}{\partial\sigma\partial w} < 0 \iff \left|\epsilon_{A',\mu}\right|\epsilon_{\mu,\sigma} < \epsilon_{VAR(Y),\sigma},$$

where $\left|\epsilon_{A',\mu}\right| \equiv \left|\frac{\mu A''(w+\mu(\sigma))}{A'(w+\mu(\sigma))}\right|$ *and* $\epsilon_{VAR(Y),\sigma} \equiv \frac{\sigma(2\sigma)}{\sigma^2} = 2$.

Proof. Applying the implicit function theorem to the first order condition, we know that $\frac{d\sigma^A}{dw}$ has the same sign as

$$\begin{aligned}\frac{\partial^2 C}{\partial\sigma\partial w} &= -\frac{\partial^2\Pi}{\partial\sigma\partial w} = \frac{1}{2}\left(2\sigma A' + A''\mu'\sigma^2\right) > 0 \\ &\iff \frac{\mu A''}{-A'}\frac{\sigma\mu'}{\mu} < \frac{\sigma\left(2\sigma\right)}{\sigma^2} = 2.\end{aligned}$$

[3] we make use of Pratt's approximation.

[4] The superscript A stands for "Autarky".

■

The necessary and sufficient condition for richer households to take higher risks is that the cross derivative of expected utility with respect to risk-taking and wealth is positive. This condition is satisfied as soon as the marginal increase in the risk premium induced by risk-taking is lower for the rich than for the poor.[5]

Higher risk-taking by the rich is widely documented and the model should replicate this result. The condition stated in Proposition 1 is indeed relatively mild, and is for instance always satisfied under CRRA preferences.

Corollary 1 *Under CRRA preferences, risk-taking is increasing with household wealth in autarky.*

Proof. With CRRA preferences, $|\epsilon_{A',\mu}| = 2\frac{\mu}{w+\mu} < 2$, and since $\epsilon_{\mu,\sigma} \leq 1$ (see footnote 1), the condition stated in Proposition 1 is always satisfied. ■

Let us now introduce community risk-sharing.

3 Risk-sharing

In addition to their decision on risk-taking, households also have the possibility to mitigate shocks ex post by sharing risks within the community. Risk pooling is achieved by income transfers between agents and takes place within multilateral relationships. Before introducing risk-sharing more formally, it is useful to enrich the description of the risk environment.

3.1 The risk environment

A crucial aspect to take into account while introducing risk-sharing is the nature of shocks. Whereas this nature was irrelevant in the case of autarky, it is needed here to distinguish between idiosyncratic and covariate shocks. Indeed, the former can be easily mitigated through transfers while the latter cannot. Let us therefore rewrite the aggregate shock faced by household h in the following way:

$$S_h = I_h + J_h,$$

where I_h and J_h respectively denote the idiosyncratic and covariate (or joint) shocks faced by household h. By definition, idiosyncratic shocks are assumed independently distributed across agents, whereas covariate shocks are perfectly correlated across agents. Conditional on risk-taking σ_h, these shocks both have a mean equal to zero and a variance of respectively $\iota\sigma_h^2$ and $\zeta\sigma_h^2$, where without loss of generality $\zeta + \iota = 1$:

$$\begin{aligned} E(I_h;\sigma_h) &= E(J_h;\sigma_h) = 0, \\ Var(I_h;\sigma_h) &= \iota\sigma_h^2, \\ Var(J_h;\sigma_h) &= \zeta\sigma_h^2 = (1-\iota)\,\sigma_h^2. \end{aligned} \tag{3}$$

[5] This is the case if marginal risk aversion A' is not too elastic to risk-taking σ compared to the consumption variance. In other words, since A'' is positive, this condition on the elasticity of A' means that the deceleration of the decreasing absolute risk aversion A shouldn't be too strong. Indeed, if risk aversion keeps on decreasing with expected consumption sufficiently, then risk-taking is more profitable for the rich.

Conditional on risk-taking σ_h, I_h and J_h are assumed independently distributed, so that this setting is consistent with that presented in the previous section:

$$\begin{aligned} E(S_h;\sigma_h) &= E(I_h;\sigma_h) + E(J_h;\sigma_h) = 0, \\ Var(S_h;\sigma_h) &= Var(I_h;\sigma_h) + Var(J_h;\sigma_h) = (\zeta + \iota)\,\sigma_h^2 = \sigma_h^2. \end{aligned}$$

It is important to note that the notion of risks that agents may want to take (and insure) is broad under this modeling strategy. Indeed, this model may be applied to a set of various income shocks, even those which are a priori neither purely covariate, nor purely idiosyncratic. For instance, weather conditions, although generally considered as covariate, are never perfectly identical among households within a same village. On the other hand, health shocks due for example to infectious diseases cannot be seen as purely idiosyncratic. In real life conditions, any given type of income shock is likely to have both an idiosyncratic and a covariate component. In our framework, the parameter ι is intended to capture the fraction of risks which is orthogonal between individuals. Actually, in any given shock, such a fraction can always be obtained by construction.

With this basic set of assumptions, we have the following structure between underlying "raw" shocks, I and J, and the way they translate into income shocks through risk-taking strategies, I_h and J_h. Let I and J be random variables with marginal distribution functions $F(I)$ and $G(J)$ and, due to independence between them, joint density $k(I,J) = f(I)\,g(J)$. I and J be are assumed to have mean zero:

$$E(I) = 0; E(J) = 0,$$

and variances of ι and $1-\iota$, respectively:

$$Var(I) = \iota; Var(J) = 1-\iota.$$

These raw shocks do not apply directly to households ; instead, the actual income shocks faced by households are shaped by their level of risk-taking. This way of formalizing uncertainty is consistent with the view that risk management strategies are, by definition, the set of decisions that affect the shape of the distribution of shocks (Dercon (2002)). This is embodied by the choice variable σ_h in this paper.

Formally, as a consequence of the distributional assumptions made on shocks above, the idiosyncratic/covariate shocks faced by agent h with risk-strategy σ_h write respectively as

$$\begin{aligned} I_h &= \sigma_h I, \\ J_h &= \sigma_h J, \end{aligned} \tag{4}$$

where all agents make an independent draw from an identical distribution $F(I)$, whereas all agents face the same draw of J in the probability distribution $G(J)$.[6] From here on, for ease of interpretation, let us consider I_h and J_h as functions which are stochastic only through their corresponding random variable, namely I and J.[7]

[6] The fact that J_h is linear in J stems from the perfect correlation between covariate shocks. The function written in (4) is the only linear function satisfying the assumptions on conditional mean and variance made in (3).

[7] All the results of this paper can be derived by treating explicitely I_h and J_h as random variables whoses distributions are conditional on σ_h as a parameter of the distributions. However, notations and interpretations are vastly simplified by focusing on I and J as the only random variables, for which we only need to account for unconditional distributions.

3.2 The risk-sharing mechanism

We are now set to introduce the concepts of risk-sharing. We model risk sharing as a two-step procedure of contribution to and then redistribution of a virtual income pool. This implies that everyone in the community is potentially exposed to the shocks faced by everyone. Even if, in the real world, transfers are likely to be decentralized, income pooling is the most convenient way of representing risk-sharing. Provided all households are linked by insurance exchanges, at least indirectly, all individual shocks are transmitted through the whole network and a community income pool is then an appropriate formalization. This condition can actually be used as the definition of our community: the community is the set of individuals who are linked directly or indirectly by insurance transfers. The composition of the community, defined as the social unit where risk-sharing takes place, is exogenously given. It is therefore aimed at representing kinship ties, for instance. This is consistent with the results of Angelucci et al. (2010) and Fafchamps & Lund (2003) according to whom risk is essentially shared within the extended family. This important assumption is further discussed later on in the paper.

In order to properly capture decentralized risk-sharing, our insurance scheme has to satisfy a series of other important requirements. This allows us to introduce the set of assumptions regarding the risk-sharing mechanism.

A1. Exogenous composition of the insurance network. This first assumption has been introduced in the preceding paragraph.

A2. Perfect commitment. As already mentioned, limited commitment has been highlighted by the literature as a major constraint that tends to restrict the scope of risk-sharing (Coate & Ravallion (1993); Ligon et al. (2002); Murgai et al. (2002); Genicot & Ray (2003)). For the sake of clarity, we assume here that insurance transfers are perfectly enforceable. This assumption could be easily relaxed. With limited commitment, an upper bound on transfers would appear in order to make them incentive compatible. This would reduce the level of subscription to mutual insurance and result in incomplete risk-sharing. However, even under perfect commitment, we are able to show that risk-sharing will be incomplete in equilibrium, thereby pointing out another source of imperfection which is, as we show below, related to wealth heterogeneity within the population. The impact of limited commitment would therefore hide the role played by wealth heterogeneity and would produce unclear predictions. As a final remark, let us mention the fact that this assumption is quite consistent with the previous one. Indeed, enforcement issues are more likely to be solved in exogenous and stable groups such as in the context of an extended family.

A3. Actuarial fairness. In order to isolate the interaction between risk managing and risk coping strategies from other considerations, we have to focus on pure risk-sharing. In order to do so, we assume that the risk-sharing mechanism does not entail income redistribution to the poor nor rent extraction by the rich. Actuarial fairness is therefore required, in the sense that, on average, households are neither net contributors nor net receivers in terms of transfers. In other words, expected transfers are zero and are just aimed at reducing the variance of consumption. The expectation of fair reciprocity is illustrated by the widespread use of quasi-credit, that is state-contingent loans (see, for instance, Udry (1994) and Thomas & Worrall (2002)). Indeed, risk-sharing transfers often takes the form of a more explicit loan contract in the sense that there is a strong and credible expectation that repayment will be made in the near future. The empirical relevance of quasi credit is a case for the relevance of actuarial fairness in the model.

A4. Subscription on an individual basis. Finally, the risk-sharing mechanisms is not characterized by a unique parameter for the whole community. In other words, risk-sharing is not governed by a social norm. Rather, we let the involvement in risk-sharing vary between agents. In the model, the decision variable $\alpha_h \in [0,1]$ will stand for agent h's level of subscription to community risk-sharing. Our aim with this variable is to highlight the interaction between risk-taking and risk-sharing decisions as well as to provide a prediction regarding the impact of wealth on those decisions.

We now turn the formal description of the risk-sharing mechanism.

Each household $h \in [0; H]$ credibly commits (A2) to provide a share $\alpha_h \in [0,1]$ of its -random- income Y_h, so that the contribution to the pool -the "transfer out"- by household h writes

$$T_{Oh} = \alpha_h Y_h, \tag{5}$$

with

$$\begin{aligned} E\left(T_{Oh}\right) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left[\alpha_h\left(\mu\left(\sigma_h\right)+\sigma_h\left(I+J\right)\right)\right] k\left(I,J\right) dIdJ \\ &= \alpha_h\mu\left(\sigma_h\right)+\alpha_h\sigma_h E(I)+\alpha_h\sigma_h E(J)=\alpha_h\mu\left(\sigma_h\right), \end{aligned} \tag{6}$$

where use has been made of the assumption of independence between I and J.

On the other hand, the household receives a "transfer in"[8] from the income pool, noted

$$T_{Ih} = r_h P, \tag{7}$$

where P is the sum of all agent's contributions gathered in the pool:

$$P \equiv \int_0^H \alpha_h Y_h dh = \int_0^H \alpha_h \mu\left(\sigma_h\right) dh + \int_0^H \alpha_h I_h dh + \int_0^H \alpha_h J_h dh.$$

Note that $\int_0^H \alpha_h I_h dh$ converges almost surely to zero by a direct application of the Strong Law of Large Numbers on independently and non-identically distributed random variables. Intuitively, the sum of all idiosyncratic shocks in the population is the sum of a continuum of independent zero-mean random variables, which tends to zero.[9] Furthermore, since $J_h = \sigma_h J$, $\int_0^H \alpha_h J_h dh = J\int_0^H \alpha_h\sigma_h dh$. As a result,

$$P \rightarrow \int_0^H \alpha_h\mu\left(\sigma_h\right)dh + J\int_0^H \alpha_h\sigma_h dh.$$

The income pool P therefore converges to the sum of individual expected contributions which remains affected by the joint shock J faced by the community. The magnitude of this shock depends on the risk-taking profile of the participants to risk-sharing. More precisely, it depends on a weighted average of the risk-taking behaviors

[8] Actually, there is obviously a unique net transfer $(T_{Oh} - T_{Ih})$ whose nature (in or out) depends on its sign.

[9] This result follows from the fact that the insurance group is represented by a continuum of individuals. The latter assumption aims at simplifying the analytical developments but does not alter our results. Indeed, if the group was of finite size, some idiosyncratic risk would remain uninsured and the effectiveness of risk-sharing would be reduced. In a model with limited commitment, Weynants (2010) precisely makes use of the latter effect to show the existence of a tradeoff in the group size between the reduction of idiosyncratic risk and the increase in enforceability issues. In addition, we can mention the work of Genicot & Ray (2003) who show the existence of an upper bound in group size in order to satisfy stability to coalition deviations.

We therefore adopt the best case scenario for idiosyncratic risk-sharing (group of infinite size) and yet we highlight incomplete subscription by some agents (see infra).

where weights are given by the participant's level of subscription to the pool. This interpretation appears more clearly if we make use of the following notations in order to define "average behaviors":

$$\bar{\alpha} \equiv \int_0^H \frac{\alpha_h}{H} dh, \tag{8}$$

$$\tilde{\sigma} \equiv \int_0^H \frac{\alpha_h}{\bar{\alpha} H} \sigma_h dh, \tag{9}$$

$$\tilde{\mu} \equiv \int_0^H \frac{\alpha_h}{\bar{\alpha} H} \mu(\sigma_h) dh. \tag{10}$$

$\bar{\alpha}$ simply denotes average subscription to risk-sharing, that is the average contribution to the income pool. $\tilde{\sigma}$ and $\tilde{\mu}$ are the weighted averages of risk-taking and expected incomes, respectively, with the relative contributions $\alpha_h/\bar{\alpha}H$ as weights. This notation is intended to capture the behavior of the representative agent in the pool. Making use of this notation, one can write

$$P \to \bar{\alpha}(\tilde{\mu} + J\tilde{\sigma}) H.$$

Since $E(J) = 0$, we have that

$$E(P) = \bar{\alpha} H \tilde{\mu}, \tag{11}$$

and since $Var(J) = 1 - \iota$,

$$Var(P) = (\bar{\alpha} H)^2 (1 - \iota) \tilde{\sigma}^2.$$

One can immediately see that, while risk-sharing allows to absorb all idiosyncratic shocks, it pools the risks resulting from covariate shocks.

It remains to provide an explicit definition of the transfer in. To this end, r_h is calculated so as to satisfy actuarial fairness (A3). The sharing rule r_h must be such that the expected contribution to the pool of each agent equals the expected transfer she receives. For all $h \in [0; H]$,

$$E(T_{Oh}) = E(T_{Ih}) \iff r_h = \frac{1}{H} \frac{\alpha_h}{\bar{\alpha}} \frac{\mu_h}{\tilde{\mu}},$$

where use has been made of equations (6), (7) and (11). The share of the income pool to which a household is entitled depends on two factors. First, it is an increasing function of the household's relative participation to risk-sharing, $\alpha_h/\bar{\alpha}$. The higher the subscription to risk-sharing as compared to the others' subscriptions, the higher the transfer received. Second, r_h is proportional to the ratio between the household's expected income, $\mu(\sigma_h)$ and the community's weighted average $\tilde{\mu}$. The higher the agent's expected income, the higher the share of the pool she receives. Those two factors are intuitively needed to obtain actuarial fairness, since higher contributors should receive more. Finally, notice that under simple income pooling, each participant to the pool would simply receive an equal share $1/H$ of the income pool.
We are now able to write the post-transfer income as

$$X_h = Y_h + (T_{Ih} - T_{Oh}) \tag{12}$$

$$= (1 - \alpha_h) Y_h + r_h P. \tag{13}$$

Let us exploit this structure in order to determine the mean and variance of consumption under risk-sharing, recalling that an agent's consumption level is the sum of her wealth and her post-transfer income: $c_h = w_h + X_h$.

Lemma 1 *The mean and variance of consumption c_h under risk-sharing write*

$$\begin{aligned} E(c_h) &= w_h + E(X_h) = w_h + \mu(\sigma_h), \\ Var(c_h) &= Var(X_h) \equiv \Sigma_h = \Sigma_{Ih} + \Sigma_{Jh} \\ &= (1-\alpha_h)^2 Var(I_h) + \left[(1-\alpha_h)\sqrt{Var(J_h)} + r_h\sqrt{Var(P)}\right]^2 \quad (14) \\ &= (1-\alpha_h)^2 \iota\sigma_h^2 + \left[(1-\alpha_h) + \alpha_h\Theta_h\right]^2 (1-\iota)\sigma_h^2, \quad (15) \end{aligned}$$

where

$$\Theta_h = \frac{\mu(\sigma_h)/\sigma_h}{\tilde{\mu}/\tilde{\sigma}}. \quad (16)$$

Proof. Provided in Appendix. ■

The first and second term on the right hand side of (14) are the idiosyncratic and covariate variances, respectively. Two interesting observations can be made. First, subscription to risk-sharing reduces the idiosyncratic risk and moreover, if the agent fully subscribes ($\alpha_h = 1$), her idiosyncratic risk vanishes. This is intuitive since idiosyncratic shocks are, by definition, orthogonal between individuals and since there is a continuum of participants. On the contrary, if the households does not participate to risk-sharing ($\alpha_h = 0$), it is easy to see that expression (15) boils down to the variance of autarky $\left(\sigma_h^2\right)$. Second, it can be seen that the agent is exposed to the variance of the income pool. The extent of the covariate risk an agent faces is not only determined by her own risk taking but also by the others' risk-taking. This exposure is proportional to the agent's subscription level. More precisely, as already highlighted, the variance of the income pool is influenced by a weighted sum of all the participants' risk-taking behaviors, where the weights are determined by each participant's level of subscription to risk-pooling. Put differently, $Var(P)$ depends on the risk-taking behavior of the representative agent in the income pool:

$$Var(P) = \left(\int_0^H \alpha_h\sigma_h dh\right)^2 (1-\iota) = (\bar{\alpha}H)^2 (1-\iota)\tilde{\sigma}^2.$$

Risk-sharing therefore entails externalities in terms of risk-taking. This important phenomenon is captured by the variable Θ_h which is the only object in the consumption variance (and utility in general) which depends on the other agents' strategies. In the following section, we show that this mechanism of covariate variance externalities is the source of the classical moral hazard result in the sense of excessive risk-taking. This phenomenon is easily illustrated in the case of an homogeneous population (in addition to homogeneous preferences, every agent is endowed with the same level of wealth) which is briefly presented as a benchmark. This has also the advantage of highlighting the impact of wealth heterogeneity on the risk-taking / risk-sharing outcome.

Before turning to this case, let us define the timing of the game with risk-sharing: households simultaneously and non-cooperatively choose their levels of risk-taking $\sigma_h \in R_+$ and subscription to risk-sharing $\alpha_h \in [0,1]$. Then, shocks are realized, transfers inside the community are made, and consumption takes place.

3.3 Risk-taking and risk-sharing in a homogeneous population

Let us assume first as a benchmark that all agents are identical in every respect, and introduce the following notations. Let α^* and σ^* denote the Nash levels of subscription to risk-sharing and risk-taking, respectively, and let α^{FB} and σ^{FB} denote the first best levels of the corresponding variables with risk-sharing. By comparing these levels, we obtain the following results.

Proposition 2 *For a homogeneous population, $\alpha^* = \alpha^{FB} = 1$, and $\sigma^A < \sigma^{FB} < \sigma^*$.*

Proof. Provided in Appendix. ■

First note that identical agents engage in full risk-sharing. This result directly follows from the assumption of perfect commitment (A2) which could be easily relaxed. However, it allows to show by contrast that complete risk-sharing does not hold under wealth heterogeneity, thereby highlighting another important source of imperfection. Second, our model illustrates the classical moral hazard result by which insured agents adopt excessive risk-taking: acting non-cooperatively, agents do not internalize the negative effect of their risk-taking on the pool variance and take excessive risks as compared to the socially optimal allocation. Technically speaking, when deciding on their level of risk-taking σ_h, agents consider Θ_h, which embodies the other players' strategy as given. Moral hazard is therefore entirely imputable to externalities in terms of covariate risk. As we show in the following section, this mechanism of covariate risk externalities has a fundamental impact on the pattern of subscription to risk-sharing under wealth heterogeneity. Finally, we see that the lack of insurance in autarky implies insufficient risk-taking.

Let us now move to the key part of the paper, namely the section which deals with the case of a heterogeneous population.

3.4 Risk-taking and risk-sharing in a heterogeneous population

This section aims at determining the Nash levels of risk-taking and subscription to risk-sharing in the case of a heterogeneous population and to highlight how both variables are affected by household wealth. In order to do so, let us calculate the first order conditions. The agents maximize $u(C_h)$ with respect to σ_h and α_h, where the certainty equivalent of consumption under risk-sharing writes

$$C_h = w_h + \mu(\sigma_h) - \frac{A(w_h + \mu(\sigma_h))\Sigma_h}{2}.$$

The first order condition with respect to σ_h is given by

$$\frac{\partial u(C_h)}{\partial \sigma_h} = 0 \iff \frac{\partial C_h}{\partial \sigma_h} = \mu'_h - \frac{1}{2}\left(A_h \frac{\partial \Sigma_h}{\partial \sigma_h} + A'_h \mu'_h \Sigma_h\right) = 0. \tag{17}$$

As in the autarkic situation, a marginal increase in individual risk-taking results in two effects: on the one hand, an increase of the income mean, which results in an increase in the expected consumption level and a decrease in absolute risk aversion, and on the other hand, an increase in the variance of consumption[10]:

$$\frac{\partial \Sigma_h}{\partial \sigma_h} = 2\sigma_h \left[\iota(1-\alpha_h)^2 + (1-\iota)(1-\alpha_h(1-\Theta_h))(1-\alpha_h(1-\Theta_h\epsilon_{\mu,\sigma}))\right] > 0. \tag{20}$$

[10] Indeed, making use of (15), the impact of risk-taking on the variance of consumption writes

$$\frac{\partial \Sigma_h}{\partial \sigma_h} = 2\sigma_h \left[\iota(1-\alpha_h)^2 + (1-\iota)\left[(1-\alpha_h) + \alpha_h\Theta_h\right]^2\right] + 2\sigma_h^2(1-\iota)\left[(1-\alpha_h) + \alpha_h\Theta_h\right]\alpha_h \frac{\partial \Theta_h}{\partial \sigma_h},$$

where, using the definition of Θ_h (16), (due to agent atomicity, $\frac{\partial \tilde{\mu}/\partial \tilde{\sigma}}{\partial \sigma_h} = 0$)

$$\frac{\partial \Theta_h}{\partial \sigma_h} = \frac{1}{\tilde{\mu}/\tilde{\sigma}}\left[\frac{\mu'(\sigma_h)\sigma_h - \mu(\sigma_h)}{\sigma_h^2}\right] = -\frac{1-\epsilon_{\mu,\sigma}}{\sigma_h}\Theta_h. \tag{18}$$

Substituting, we end up with

$$\frac{\partial \Sigma_h}{\partial \sigma_h} = 2\sigma_h \left[\iota(1-\alpha_h)^2 + (1-\iota)(1-\alpha_h(1-\Theta_h))(1-\alpha_h(1-\Theta_h\epsilon_{\mu,\sigma}))\right] > 0. \tag{19}$$

The above equation shows that an increase in risk-taking unambiguously increases both the idiosyncratic and covariate parts of the income variance. As will be shown below, the household balances both effects on expected income and risk premium depending on its wealth level w and its subscription to risk-sharing.

Let us now analyze the agent's reaction function in terms of subscription to risk-sharing. First note that, by actuarial fairness (A3), subscription to risk-sharing does not impact on expected consumption: $\partial\mu/\partial\alpha_h = 0$. Therefore, the decision on α_h only depends on its effect on the variance of consumption Σ_h. The latter can be usefully decomposed into the impact of an increase in α_h on idiosyncratic and covariate risks, respectively:

$$\frac{\partial\Sigma_h}{\partial\alpha_h} = \frac{\partial\Sigma_{Ih}}{\partial\alpha_h} + \frac{\partial\Sigma_{Jh}}{\partial\alpha_h},$$

where, making use of (15)

$$\begin{aligned}
\frac{\partial\Sigma_{Ih}}{\partial\alpha_h} &= -2\left(1-\alpha_h\right)\iota\sigma_h^2 < 0, \\
\frac{\partial\Sigma_{Jh}}{\partial\alpha_h} &= 2\left[\left(1-\alpha_h\right)+\alpha_h\Theta_h\right]\left(1-\iota\right)\sigma_h^2\left(\Theta_h-1\right) < 0 \\
&\iff \Theta_h < 1 \iff \frac{\mu\left(\sigma_h\right)}{\sigma_h} < \frac{\tilde{\mu}}{\tilde{\sigma}}.
\end{aligned}$$

It appears, on the one hand, that the idiosyncratic risk always decreases with higher risk-sharing. However, on the other hand, the impact of subscription to risk-sharing on the covariate variance critically depends on the household's risk-taking level relative to the behavior of the representative agent in the income pool. Formally, the agent's first order condition with respect to risk-sharing can only be satisfied with equality if there is a tradeoff between the reduction of the idiosyncratic risk and an increase in the covariate risk. In the latter case, the optimal subscription level will be interior, that is incomplete. Otherwise, the household will be at a corner, that is will opt for full subscription. This result gives rise to the following Lemma:

Lemma 2 *Agents taking low risks only partially subscribe to risk-sharing:* $\alpha_h^* = \frac{\iota-(1-\iota)(\Theta_h-1)}{\iota+(1-\iota)(\Theta_h-1)^2} < 1$ *if and only if* $\sigma_h < \hat{\sigma}$. *Otherwise,* $\alpha_h^* = 1$.

Proof. First, we have that

$$\frac{\partial\Sigma_h}{\partial\alpha_h} = 0 \iff \alpha_h^* = \frac{\iota-\left(1-\iota\right)\left(\Theta_h-1\right)}{\iota+\left(1-\iota\right)\left(\Theta_h-1\right)^2} < 1 \iff \Theta_h > 1.$$

Second, let $h\left(\sigma\right) \equiv \mu\left(\sigma\right)/\sigma$, with $h'\left(\sigma\right) = \frac{\mu'\sigma-\mu}{\sigma^2} < 0$ since[11] $\frac{\mu'\sigma}{\mu} \equiv \epsilon_{\mu,\sigma} < 1$, and let $\hat{\sigma} \equiv h^{-1}\left(\tilde{\mu}/\tilde{\sigma}\right)$. Therefore, recalling that

$$\Theta_h = \frac{\mu\left(\sigma_h\right)/\sigma_h}{\tilde{\mu}/\tilde{\sigma}},$$

$\Theta_h = h\left(\sigma_h\right)/h\left(\hat{\sigma}\right) > 1$ if and only if $\sigma_h < \hat{\sigma}$. ■

Individuals taking more risks than $\hat{\sigma}$ benefit from risk-sharing through a reduction of both idiosyncratic and covariate variances. In contrast, individuals taking less risks than $\hat{\sigma}$ face a tradeoff using α. This tradeoff balances the reduction of the idiosyncratic variance and the increase in the covariate variance. As previously

[11] As already mentioned, the three assumption on $\mu\left(\sigma\right)$ imply that the average return to risk taking is always larger than the marginal return:

$$\frac{\mu}{\sigma} \geq \mu' \iff \epsilon_{\mu,\sigma} \leq 1.$$

described, the adverse effect on covariate variance is due to a negative externality generated by agents taking aggressive risks, which pollute the pool via covariate shocks.

Let us now introduce the main proposition of the paper, which rationalizes the twofold stylized facts that rich farm-households are higher risk-takers whereas they are less vulnerable to income shocks thanks to a higher subscription to risk-sharing.

Proposition 3 *Risk-taking σ_h^* and risk-sharing α_h^* are both increasing in household wealth w_h if and only if*

$$\frac{\partial^2 \Pi}{\partial \sigma \partial w} < 0 \iff |\epsilon_{A',\mu}|\, \epsilon_{\mu,\sigma} < \epsilon_{\Sigma,\sigma}.$$

Rich agents whose risk-taking σ_h^* is larger than $\hat{\sigma}$ fully subscribe to risk-sharing ($\alpha_h^* = 1$), whereas poorer agents, whose risk-taking is smaller than $\hat{\sigma}$, only partially subscribe to risk-sharing ($\alpha_h^* = \frac{\iota-(1-\iota)(\Theta_h-1)}{\iota+(1-\iota)(\Theta_h-1)^2} < 1$).

Proof. Provided in Appendix. ∎

Two elements are worth pointing out.

First, it appears that wealth inequalities result in incomplete risk-sharing, even under perfect commitment (A2). This can be seen by comparing Proposition 2 with Proposition 3. Indeed, under the same set of assumptions, there is complete risk-sharing within a homogeneous group. This is the outcome of the mechanism of covariate risk externalities that our model allows to highlight. The residual variance of the income pool is affected by the entire risk-taking profile. Besides, wealth heterogeneity makes some agents relatively more (and other agents less) able to cope with risk ex post. The equilibrium risk-taking profile is therefore asymmetric. As a result, while every participant to risk-pooling benefits from a reduction of her idiosyncratic risk, some agents suffer from an increase in the magnitude of joint shocks. This provides them with an incentive to subscribe only partially to risk-sharing in order to reach the optimal balance between both effects. While the literature generally points out limited commitment as the source of incomplete risk-sharing, we highlight wealth heterogeneity as another potential explanation.

Second, in the light of the above result, one could argue that, on the one hand, homogenous insurance groups are desirable since the level of risk-sharing achieved in these groups is higher and that, on the other hand, our model prevents such groups from emerging in equilibrium since group composition is exogenously given. This paper provides indeed, at least indirectly, a rationale for the existence of insurance groups whose participants share similar characteristics. However, the fact that homogeneous groups share risk more efficiently remains unclear. Indeed, recall that, on the one hand, Proposition 2 shows that, in homogenous groups, complete risk-sharing result in moral hazard, that is excessive risk-taking. On the other hand, if one relaxes the assumption of perfect commitment, the latter effect could be mitigated since risk-sharing would also be incomplete. In addition, it can also be argued that our prediction remains valid even within quite homogeneous group since some residual differences between participants are likely to remain. A deeper analysis of this question would require empirical investigations. Finally, we argue that some insurance networks are by nature of exogenous composition, such as kinship ties.

4 From wealth to risk coping

In addition to the household's decisions on risk-taking and subscription to risk-sharing, the use of buffer stocks is a third important strategy to be taken into account. If insurance and credit markets are missing, wealth is a major determinant of the household's ability to smooth consumption ex post. This section simply aims at showing that the analysis conducted up to now is valid as a reduced form of a more complete version of the game where, in a second period, a decision is made on the allocation of wealth between current consumption and buffer stocks kept for subsequent periods. More precisely, after uncertainty is realized and risk-sharing transfers have been made, the agent chooses how much to use as a buffer b, so that consumption now writes $c = X + b$. The rest of the wealth $i \equiv w - b$ has therefore a twofold purpose: it is left as a means of facing future shocks but can also be seen as the amount of inheritance for the household's children. In order to define the optimal use of buffer stocks after the shock's realization, we solve the following optimization program

$$\underset{b \in [-X,w]}{Max} U = u(X+b) + v(w-b),$$

where $v(.)$ denotes either indirect (expected) utility of the subsequent periods or a term of altruism towards the children. The optimal level of buffer b^* is implicitly defined by the first order condition

$$\frac{\partial U}{\partial b} = u'(X+b^*) - v'(w-b^*) = 0. \tag{21}$$

This condition gives rise to the following proposition:

Proposition 4 *The optimal use of buffer stock increases with wealth w and decreases with the post-transfer income X: $\frac{db^*}{dw} > 0$, $\frac{db^*}{dX} < 0$.*

Proof. Simply, making use of the implicit function theorem,

$$\frac{db^*}{dw} > 0 \iff \frac{\partial^2 U}{\partial b \partial w} = -v''(w-b^*) > 0,$$

which always holds.

Following the same reasoning,

$$\frac{db^*}{dX} < 0 \iff \frac{\partial^2 U}{\partial b \partial X} = u''(X+b^*) < 0.$$

■

Note that ex-ante, the use of buffer stocks also becomes a random variable, which depends, under risk-sharing, on the realization of the shocks of the whole community. In order to complete the analysis and identify the equilibrium levels of risk-taking and risk-sharing, we therefore need to maximize

$$EU = Eu(X + b^*(X,w)) + Ev(w - b^*(X,w)).$$

In order to keep the resolution tractable, we assume CRRA utility functions from here on, that is, $u(c) = c^{1-a}/(1-a)$ and $v(c) = \gamma c^{1-a}/(1-a)$. In this case[12], equation (21) implies

$$b^* = w\beta - X(1-\beta),$$

[12] A strict interpretation of this setting could also be that there exist a final period during which the agent does not work anymore (and therefore faces no uncertainty) and consumes her residual wealth.

where

$$\beta = \frac{\gamma^{-\frac{1}{a}}}{1+\gamma^{-\frac{1}{a}}} \in [0;1] .$$

As a result,

$$\begin{aligned} c &= \beta (X + w) , \\ i &= (1-\beta)(X+w) . \end{aligned}$$

Analyzing now the decision prior to the realization of shocks, let us note that expected utility writes

$$\begin{aligned} EU &= Eu(\beta (X+w)) + \gamma Eu((1-\beta)(X+w)) \\ &= \left(\beta^{1-a} + \gamma (1-\beta)^{1-a}\right) E\left[\frac{(X+w))^{1-a}}{1-a}\right] \\ &= \left(1+\gamma^{\frac{1}{a}}\right)^{a} E\left[\frac{(X+w)^{1-a}}{1-a}\right] . \end{aligned}$$

In other words, under CRRA, maximizing utility with endogenous buffer stocks just implies a rescaling of the utility function by the constant $\left(1+\gamma^{\frac{1}{a}}\right)^{a}$. Therefore, compared to the previous section, the whole game is unchanged and results are strictly identical.

5 Conclusion

The analysis conducted in this paper has been motivated by a twofold empirical observation: on the one hand, poor farm households are reluctant to adopt modern technologies characterized by high levels of risk and return. On the other hand, they tend to suffer from a higher exposure to idiosyncratic shocks, indicating a lower involvement in risk-sharing mechanisms. These stylized facts appear as the outcome of a model in which risk-taking and risk-sharing decisions are non-cooperatively taken in insurance groups that are characterized by some degree of wealth heterogeneity.

As the first step of our model illustrates, production decisions are taken in the face of the basic tradeoff between risk and return. As intuition suggests, wealth is an important determinant of the ability to smooth consumption ex post, once uncertainty is revealed. Wealthier agents are therefore endowed with an additional instrument do deal with risk that allows them to take advantage of more profitable production choices.

The positive relationship between wealth and risk-taking is maintained and even reinforced once a risk-pooling mechanism is introduced. Indeed, we show that the higher the risk adopted ex ante as compared to the risk-taking behavior of the representative agent in the insurance group, the more the household benefits from risk-sharing. This result stems from the fact that the income pool, while allowing to absorb the idiosyncratic fraction of the shocks faced by participants, remains affected by their covariate component. This distinction between idiosyncratic, that is orthogonal, and covariate risks is at the same time fundamental from a conceptual point of view and empirically relevant since any given shock is likely to have both characteristics. Besides, risk management strategies such as production and technological choices shape the distribution of any type of shock in a similar fashion in the sense that the magnitude of the shock, idiosyncratic or covariate, is always positively related to the extent of risk-taking. It follows that the variance of the income pool depends on the whole risk-taking profile. Subscription to the pool therefore entails an exposure to the other participants' risk-taking behavior.

In heterogeneous groups, this externality can explain partial subscription to risk-sharing by the poor and their resulting lack of protection against idiosyncratic risk. We have also highlighted that incomplete risk-sharing could occur even under perfect commitment and that wealth heterogeneity was an alternative source of explanation. In addition, in homogeneous groups, this externality is the source of moral hazard in the sense of excessive risk-taking.

To conclude this paper, it can be noted that our theoretical prediction is quite pessimistic with respect to poverty traps issues. Indeed, since poverty is directly linked to the ability to cope with risk ex post, the model predicts that the lower is this ability, the less the household is involved in risk-sharing. As a consequence, ex post means of insurance are weak on the aggregate and poor household have to rely on ex ante self-insurance mechanisms. Hence, their production decisions are highly affected by risk considerations which make them reluctant to adopt the more profitable technologies.

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6 Appendix

6.1 Proof of Lemma 1

Proving the first part of the lemma is straightforward. Indeed, since the mutual insurance mechanism is actuarially fair,

$$\begin{aligned} E(X_h) &= E(Y_h) - E\left(T_{Oh}\right) + r_h E\left(P\right) \\ &= E(Y_h) = \mu\left(\sigma_h\right). \end{aligned}$$

Proving the second part of the Lemma is more requiring. Making use of equation (13), we can write

$$\begin{aligned} Var(c_h) &= Var((1-\alpha_h)\,Y_h + r_h P) && (22) \\ &= (1-\alpha_h)^2\,Var(Y_h) + r_h^2 Var(P) + 2\,(1-\alpha_h)\,r_h Cov(Y_h, P). && (23) \end{aligned}$$

Let us treat each of these terms separately.
The first term of (23) is $(1-\alpha_h)^2\,Var(Y_h)$, which is by definition $(1-\alpha_h)^2\,\sigma_h^2$.
The second term of (23) is somewhat more demanding. It is based on the variance of the insurance pool, which, as was shown above, writes

$$Var\left(P\right) = \left(\int_0^H \alpha_h \sigma_h dh\right)^2 (1-\iota)\,,$$

Third,

$$\begin{aligned} Cov(Y_h, P) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left[Y_h - E\left[Y_h\right]\right]\left[P - E\left[P\right]\right] k\left(I, J\right) dI dJ \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left[\sigma_h\left(I+J\right)\right]\left[J\int_0^H \alpha_h\sigma_h dh\right] k\left(I,J\right) dIdJ \\ &= \sigma_h \int_0^H \alpha_h\sigma_h dh \left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[IJ\right] f\left(I\right) g\left(J\right) dIdJ + \int_{-\infty}^{+\infty} J^2 g\left(J\right) dJ\right] \\ &= (1-\iota)\,\sigma_h \int_0^H \alpha_h\sigma_h dh. \end{aligned}$$

As a result,

$$\begin{aligned} Var(c_h) &= (1-\alpha_h)^2\,\sigma_h^2 + r_h^2\,(1-\iota)\left[\int_0^H \alpha_h\sigma_h\right]^2 + 2\,(1-\alpha_h)\,r_h\,(1-\iota)\,\sigma_h \int_0^H \alpha_h\sigma_h dh \\ &= \iota\,(1-\alpha_h)^2\,\sigma_h^2 + (1-\iota)\left[(1-\alpha_h)^2\,\sigma_h^2 + r_h^2\left[\int_0^H \alpha_h\sigma_h\right]^2 + 2\,(1-\alpha_h)\,r_h\sigma_h \int_0^H \alpha_h\sigma_h dh\right] \\ &= (1-\alpha_h)^2\,\iota\sigma_h^2 + (1-\iota)\left[(1-\alpha_h)\,\sigma_h + r_h\int_0^H \alpha_h\sigma_h dh\right]^2 \\ &= (1-\alpha_h)^2\,Var\left(I_h\right) + \left[(1-\alpha_h)\sqrt{Var\left(J_h\right)} + r_h\sqrt{Var(P)}\right]^2. \end{aligned}$$

6.2 Proof of Proposition2

Let us start by characterizing (α^*, σ^*). Combining all -identical- reaction functions, agents will all opt for the same non-cooperative level of α^* and σ^*, so that,$\mu(\sigma_h)/\sigma_h = \tilde{\mu}/\tilde{\sigma}$, that is, $\Theta_h = 1$. As a result, equation (15) boils down to

$$\frac{\partial \Sigma}{\partial \alpha} = -2\sigma^{*2}\iota\left[1-\alpha^*\right] = 0 \Leftrightarrow \alpha^* = 1.$$

Making use of $\alpha_h^* = 1$, equation (19) boils down to

$$\frac{\partial \Sigma}{\partial \sigma} = 2\sigma\left(1-\iota\right)\epsilon_{\mu,\sigma},$$

and

$$\Sigma^* = \left(1-\iota\right)\sigma_h^2.$$

As a result, using (17), the non-cooperative level of risk-taking under the homogeneous population is implicitly determined by

$$\mu'\left(1+|A'|\frac{\left(1-\iota\right)\sigma^{*2}}{2}\right) - A\left(1-\iota\right)\sigma^*\epsilon_{\mu,\sigma} = 0. \tag{24}$$

Let us now analyze the first-best allocation. Here, the social planner precludes all strategic interactions, and identical agents are assigned identical behaviors. Consequently, before optimizing, $\Theta_h = 1$ for all h in the first-best. This implies that

$$\Sigma^{FB} = \sigma_h^2\left[\iota\left[1-\alpha_h\right]^2 + \left(1-\iota\right)\right].$$

Maximizing utility with respect to α, one obtains $\alpha^{FB} = 1$. The first-best level of risk-taking under the homogeneous population is implicitly determined by the first order condition, which, after simplifications, writes

$$\mu'\left(1+|A'|\frac{\left(1-\iota\right)\sigma^{FB2}}{2}\right) - A\left(1-\iota\right)\sigma^{FB} = 0. \tag{25}$$

With (2), (24) and (25), we therefore have the implicit solutions to σ^A, σ^* and σ^{FB}. Note that (2) would be identical to (25) if ι was equal to zero. Therefore, applying the implicit function theorem to (25) with respect to ι, we have that $\sigma^{FB} > \sigma^A$ since

$$\frac{d\sigma^{FB}}{d\iota} = -\frac{\frac{\partial^2 U(C)}{\partial \sigma^{FB}\partial \iota}}{\frac{\partial^2 U(C)}{\partial \sigma^{FB2}}} > 0,$$

since

$$\frac{\partial^2 U\left(C\right)}{\partial \sigma^{FB}\partial \iota} = -\frac{\partial^2 \Pi^{FB}}{\partial \sigma \partial \iota} = -\frac{\partial^2 \frac{A(w+\mu(\sigma))\sigma_h^2(1-\iota)}{2}}{\partial \sigma \partial \iota} = \frac{\partial \frac{A(w+\mu(\sigma))\sigma_h^2}{2}}{\partial \sigma} = \frac{\partial \Pi^A}{\partial \sigma} > 0.$$

Applying the same reasoning, note that (25) would be identical to (24) if $\epsilon_{\mu,\sigma}$ was equal to one. Therefore, applying the implicit function theorem to (24) with respect to $\epsilon_{\mu,\sigma}$, we have that $\sigma^{FB} < \sigma^*$ since

$$\frac{d\sigma^*}{d\epsilon_{\mu,\sigma}} = -\frac{\frac{\partial^2 U(C)}{\partial \sigma^{FB}\partial \epsilon_{\mu,\sigma}}}{\frac{\partial^2 U(C)}{\partial \sigma^{*2}}} < 0,$$

since

$$\frac{\partial^2 U\left(C\right)}{\partial \sigma^{FB}\partial \epsilon_{\mu,\sigma}} = -A\left(1-\iota\right)\sigma^* < 0.$$

6.3 Proof of Proposition 3

We need to prove that w has a positive effect on both α_h^* and σ_h^* for both agents at the corner and at the interior solution on α_i^*. The effect of w for agents at a corner solution on α_h^* is simply obtained by the single-equation implicit function theorem applied to the first order condition on σ. This is due to the fact that since α_h^* is at a corner, a marginal variation in w has no impact on α and only σ will react to w. This analysis is the same as that discussed in the proof of Proposition 1 for the case of autarky, where $Var(Y_h)$ just needs to be replaced by Σ_h, but both are positively affected by σ and unaffected by w.

Finally, let us analyze the effect of w for agents who are at an interior α_h^*. The pair of equilibrium conditions for (σ_h^*, α_h^*) is

$$\begin{pmatrix} \frac{\partial C}{\partial \sigma} \\ \frac{\partial C}{\partial \alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{26}$$

Let us use the bivariate version of the implicit function theorem on this pair of equations in order to determine the sign of $\begin{pmatrix} \frac{d\sigma_h^*}{dw} \\ \frac{d\alpha_h^*}{dw} \end{pmatrix}$. Applying the formula, we have:

$$\begin{pmatrix} \frac{d\sigma_h^*}{dw} \\ \frac{d\alpha_h^*}{dw} \end{pmatrix} = -\begin{pmatrix} \frac{\partial^2 C}{\partial \sigma^2} & \frac{\partial^2 C}{\partial \sigma \partial \alpha} \\ \frac{\partial^2 C}{\partial \alpha \partial \sigma} & \frac{\partial^2 C}{\partial \alpha^2} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial^2 C}{\partial \sigma \partial w} \\ \frac{\partial^2 C}{\partial \alpha \partial w} \end{pmatrix}.$$

First, note that

$$\begin{aligned} \begin{pmatrix} \frac{\partial^2 C}{\partial \sigma^2} & \frac{\partial^2 C}{\partial \sigma \partial \alpha} \\ \frac{\partial^2 C}{\partial \alpha \partial \sigma} & \frac{\partial^2 C}{\partial \alpha^2} \end{pmatrix}^{-1} &= \Omega^{-1} \begin{pmatrix} -\frac{1}{2} A \frac{\partial^2 \Sigma}{\partial \alpha \partial \alpha} & \frac{1}{2} A \frac{\partial^2 \Sigma}{\partial \alpha \partial \sigma} + \frac{1}{2} A' \mu' \frac{\partial \Sigma}{\partial \alpha} \\ \frac{1}{2} A \frac{\partial^2 \Sigma}{\partial \alpha \partial \sigma} + \frac{1}{2} A' \mu' \frac{\partial \Sigma}{\partial \alpha} & -\frac{1}{2} A'' \Sigma \mu'^2 - A' \frac{\partial \Sigma}{\partial \sigma} \mu' - \frac{1}{2} A \frac{\partial^2 \Sigma}{\partial \sigma \partial \sigma} + \mu'' \left(1 - \frac{1}{2} A' \Sigma\right) \end{pmatrix} \\ &= \Omega^{-1} \begin{pmatrix} -\frac{1}{2} A \frac{\partial^2 \Sigma}{\partial \alpha \partial \alpha} & \frac{1}{2} A \frac{\partial^2 \Sigma}{\partial \alpha \partial \sigma} \\ \frac{1}{2} A \frac{\partial^2 \Sigma}{\partial \alpha \partial \sigma} & -\frac{1}{2} A'' \Sigma \mu'^2 - A' \frac{\partial \Sigma}{\partial \sigma} \mu' - \frac{1}{2} A \frac{\partial^2 \Sigma}{\partial \sigma \partial \sigma} + \mu'' \left(1 - \frac{1}{2} A' \Sigma\right) \end{pmatrix}, \end{aligned}$$

with

$$\Omega = \left(\frac{\partial^2 C}{\partial \sigma^2} \frac{\partial^2 C}{\partial \alpha^2} - \left(\frac{\partial^2 C}{\partial \alpha \partial \sigma} \right)^2 \right),$$

is the determinant of the Hessian of C, and is positive for C to be at a maximum in (σ^*, α^*). Also,

$$\begin{aligned} \begin{pmatrix} \frac{\partial^2 C}{\partial \sigma \partial w} \\ \frac{\partial^2 C}{\partial \alpha \partial w} \end{pmatrix} &= \begin{pmatrix} -\frac{1}{2} A' \frac{\partial \Sigma}{\partial \sigma} - \frac{1}{2} A'' \Sigma \mu' \\ -\frac{1}{2} A' \frac{\partial \Sigma}{\partial \alpha} \end{pmatrix} \\ &= \begin{pmatrix} -\frac{1}{2} \left(A' \frac{\partial \Sigma}{\partial \sigma} + \frac{1}{2} A'' \Sigma \mu' \right) \\ 0 \end{pmatrix}, \end{aligned}$$

since $\frac{\partial \Sigma}{\partial \alpha} = 0$ at equilibrium. For the solution to be a maximum, Ω needs to be positive, with both $\frac{\partial^2 C}{\partial \sigma^2}$ and $\frac{\partial^2 C}{\partial \alpha^2}$ negative. We finish the proof of the result:

$$\begin{aligned} \begin{pmatrix} \frac{d\sigma_h^*}{dw} \\ \frac{d\alpha_h^*}{dw} \end{pmatrix} &= \frac{A\Omega^{-1}}{4} \left(A' \frac{\partial \Sigma}{\partial \sigma} + A'' \Sigma \mu' \right) \begin{pmatrix} -\frac{\partial^2 \Sigma}{\partial \alpha^2} \\ \frac{\partial^2 \Sigma}{\partial \alpha \partial \sigma} \end{pmatrix}, \\ SIGN \begin{pmatrix} \frac{d\sigma_h^*}{dw} \\ \frac{d\alpha_h^*}{dw} \end{pmatrix} &= SIGN \begin{pmatrix} -\frac{1}{4} A \frac{\partial^2 \Sigma}{\partial \alpha^2} \left(A' \frac{\partial \Sigma}{\partial \sigma} + A'' \Sigma \mu' \right) \\ \frac{1}{4} A \frac{\partial^2 \Sigma}{\partial \alpha \partial \sigma} \left(A' \frac{\partial \Sigma}{\partial \sigma} + A'' \Sigma \mu' \right) \end{pmatrix}. \end{aligned}$$

We therefore need to determine the signs of three elements, namely $\frac{\partial^2 \Sigma}{\partial \alpha^2}$, $\frac{\partial^2 \Sigma}{\partial \alpha \partial \sigma}$ and $\left(A' \frac{\partial \Sigma}{\partial \sigma} + A'' \Sigma \mu' \right)$. Since we are analyzing the interior solution in α, the second order condition in α must be satisfied. This is the case if and only if $\frac{\partial^2 \Sigma}{\partial \alpha^2} > 0$:

$$\begin{aligned}
\frac{\partial^2 u(C)}{\partial \alpha_h^2} &= u''(C) \left(\frac{A}{2} \overbrace{\frac{\partial \Sigma_h}{\partial \alpha_h}}^{=0} \right)^2 - \frac{A}{2} u'(C) \frac{\partial^2 \Sigma_h}{\partial \alpha_h^2} \\
&= -\frac{A}{2} u'(C) \frac{\partial^2 \Sigma_h}{\partial \alpha_h^2} \\
&< 0 \iff \frac{\partial^2 \Sigma_h}{\partial \alpha_h^2} > 0.
\end{aligned}$$ [13]

Making use of the fact that at the interior solution, $\frac{\partial \Sigma}{\partial \alpha} = 0$, one can show that the cross derivative is always negative:

$$\begin{aligned}
\frac{\partial^2 \Sigma}{\partial \alpha \partial \sigma} &= -2 (1 - \iota) \frac{\mu(\sigma_h)}{\tilde{\mu}/\tilde{\sigma}} (1 - \epsilon_{\mu,\sigma}) (1 + 2\alpha (\Theta_h - 1)) \\
&< 0.
\end{aligned}$$

We therefore know that the impact of w has the same sign on both σ_i^* and α_i^*. This sign is determined by $-\left(A' \frac{\partial \Sigma}{\partial \sigma} + A'' \Sigma \mu'\right)$. Equivalently,

$$\begin{pmatrix} \frac{d\sigma_i^*}{dw} \\ \frac{d\alpha_i^*}{dw} \end{pmatrix} > \begin{pmatrix} 0 \\ 0 \end{pmatrix} \iff \frac{\mu A''}{-A'} \frac{\sigma \mu'}{\mu} < \frac{\sigma \frac{\partial \Sigma}{\partial \sigma}}{\Sigma} \iff \left| \epsilon_{A',\mu} \right| \epsilon_{\mu,\sigma} < \epsilon_{\Sigma,\sigma}.$$

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